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CASE REPORT

Strategic planning of empty container repositioning in the transpacific market: a case study

Yu Zhang^{a*} and Cristiano Facanha^b

^aDepartment of Civil and Environmental Engineering, University of South Florida, 4202 E. Fowler Ave., ENB118, Tampa, FL 33620, USA; ^bInternational Council of Clean Transportation, One Post Street Suite 2700, San Francisco, CA 94104, USA

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Trade imbalances between the USA and other regions of the world, especially Asian countries, have worsened the situation, and the intermodal coordination of empty containers has become a very challenging issue for shipping lines. The topic represents a very complex logistics problem with practical applications. This analysis focused on one of the top seven shipping companies globally that is particularly strong on the transpacific trade market. The main goal was to model the operations of the company and introduce new strategies to reposition empty containers more efficiently. The concept of rail terminals should be implemented to provide economies of scale to reduce costs. Ports on the East Coast, which are not utilised to transport empty containers back to Asia, also played a role in the strategies considered in this study. The results also indicated that the use of a dedicated fleet of trucks to handle local transportation would provide significant cost savings. Based on the results, all three strategies – use of East Coast ports, use of terminals, and use of dedicated fleet – provide significant cost savings (11.3%, 15.2%, and 15.4%, respectively). Further analysis is necessary to assess practical implications of such strategies as well as to improve the proposed model.

Keywords: logistics; repositioning of empty containers; shipping line; operational strategy

1. Background

One of the major trade problems that has arisen during the last few years, especially for the transpacific market, is the container imbalances between US imports and exports. Their interference with two-way operations has affected the finances of most shipping companies. The transportation, handling, and storage of empty containers have augmented carriers' overhead costs. The effects are even being felt in Asia, where the internal movement of containers has also suffered (Hanscom 1998). The economic growth in China has greatly contributed to this problem. During the last few years, China has rapidly increased its exports to the USA (becoming the first source of US imports) while their imports have increased at a slower pace.

The imbalance of container flow from Asia to the USA has increased from 0.5 million in 1995 to 10 million in 2007 (Rodrigue 2013). Shipping costs related to the water portion of the trip are waived, as ships have the extra capacity to carry the empty containers back to Asia. Repositioning costs are mostly composed of transportation (road and rail), handling (mainly at ports, but also at

^{*}Corresponding author. Emails: yuzhang@usf.edu, yuzhang@eng.usf.edu

rail terminals), and inventory costs. The transpacific flow of empty containers is handled through four ocean terminals on the US West Coast – Seattle, Portland, Oakland, and Los Angeles.

An average container has a 10-year life span, so every year part of the container fleet is renovated while old containers are displaced. Most containers are purchased in China; therefore, new containers do not have to be repositioned. By the same token, displaced containers generally stay in North America in order to avoid repositioning costs.

2. Current operating plan

The selected shipping line goes through an annual 'fleet sizing' exercise, which determines the containers that will be replaced and the number of new containers depending on demand forecasts. Containers can be of five different types: dry 20" and dry 40", which represent the fleet majority, and dry 40" (high cube), dry 45", and reefer 40". On a weekly basis, a 'container balancing analysis' is performed. Each location in the USA reports the number of containers available (supply of empty containers) and the number of containers needed (demand for empty containers). Because shippers need to book containers a few weeks prior to departure, demand for empty containers is known. This demand comes from both Asian and North American locations.

Based on transit time and rate matrices, each empty container is allocated to a point of demand (Figure 1). Priority is given to fulfill the demand of North American locations, as containers should be loaded as often as possible. Containers that cannot be allocated to North American loads are routed to a West Coast port (Seattle, Portland, Oakland, or Los Angeles) based on the lowest cost, on their way to Asia. As volumes do not change significantly from week to week, this procedure is not always used. Instead, the same pairs are usually selected, meaning that specific locations always supply empty containers to other specific locations.

In the process of repositioning empty containers, capacity constraints such as railroad equipment, railroad power, labour at ports, and drayage equipment need to be considered. As full containers always have priority over empty containers, rail transit times are not always consistent



Figure 1. Allocation method for empty containers.



Figure 2. Container life cycle.

(e.g. 'empties' are left behind). Even though westbound vessels have extra capacity in terms of number of containers, there might be weight constraints due to the heavier nature of eastbound loads (e.g. metal scrap, garbage, and other heavy goods).

2.1. Container life cycle

With respect to a container life cycle, it is first filled with exports from Asia and then travels by ship to the USA. It is then sent from the arrival port in the USA to a customer by a combination of truck and rail. Part of the empty containers will be repositioned within the USA to satisfy the demand of containers bound to Asia, while the remaining empty containers will need to be relocated back to Asia (Figure 2).

2.2. Baseline costs

In the process of repositioning an empty container, transportation, storage, and handling costs are incurred. An empty container is also sorted on ships and at rail yards. However, compared to loaded containers that have specific destination, empty containers of the same type can substitute for each other, reducing sorting costs dramatically.

The transportation cost of repositioning empty containers includes drayage cost from the customer location to the origin rail terminal and from the origin rail terminal to the destination rail terminal, and drayage to the seaport. When the distance is short enough so that using a truck is more efficient, the containers can be transported directly to a seaport on a truck. Compared to loaded containers, rail and truck rates are slightly lower for empty containers. In the negotiation with truck companies, ocean carriers always take advantage of their high volumes to pursue lower rates. Ocean carriers usually ignore the transportation cost of empty container on their ships because, due to the imbalance between Asia and the USA, there is always extra space on ships to Asia.

Usually, handling costs at rail terminals or truck terminals are included in the transportation rate (as in our baseline cost calculation). For this analysis, we wanted to test the sensitivity of each cost for strategies to be proposed, so we extracted the handling cost from the rail rate or truck rate and considered them separately. Handling costs at seaports were not considered, as that was outside the scope of this project.

If compared to transportation and handling costs, storage costs are almost negligible, especially in the storage area out of seaports due to lower rents. Ocean carriers rent a certain amount of space

Total feus repositioned to Asia	140,409
Total feus repositioned within the USA	297,140
Trucking costs	\$74,567,838
Rail costs	\$73,051,908
Inventory costs	\$3,413,542
Handling costs	\$9,223,808
Terminal costs	-
Total cost	\$160,257,096

Table 1. Estimation of baseline costs.

at seaports with a static rate according to their volume. If they want extra space, they negotiate with seaport at another rate. As previously mentioned, empty containers of the same type can substitute each other, so the average storage time of an empty container is very short.

Baseline costs were estimated to calculate transportation costs. Due to the purpose of business confidentiality, the data provided by the shipping company were not the most current information. Nevertheless, this did not affect the demonstration of the proposed methodology and relative comparison of different strategies, which are elaborated in later sections. Note that corresponding cost parameters for the same year are used in the calculation that may not reflect the present situation. It was assumed that about 31% of the containers that move directly to Asia (without repositioning in the USA) originate on states on the US West Coast (California, Washington, and Oregon). Due to the short average distance from customer locations to seaports, it is reasonable to assume that empty containers are transported by trucks. In total, 55% of the demand for empty containers originates on the US West Coast. For empty containers on the West Coast, the average truck distance to a seaport is 300 miles. For the repositioned containers on the West Coast, the average distance is 200 miles. The remaining volume originates in the other US states. We assumed that such volume is transported by rail. The average drayage distance from rail terminals to customer locations is always 75 miles, while the average rail distance is 2000 miles. If they are repositioned in the USA, the average rail distance is 500 miles. To calculate inventory costs (costs due to capital investment on containers), the following assumptions were made: containers are worth \$2000 and have a lifetime of 10 years, the interest rate is 7% per year, and each container is empty for 10 days. Based on this information, baseline costs could be estimated (Table 1).

3. Literature review

A problem similar to the repositioning of empty container is empty vehicle redistribution. Literature in that area can be traced back to the 1970s. Assuming that decision-makers have complete information, Leddon and Wrathall (1967) and White (1972) developed deterministic models for railcar distribution. In the line of stochastic environments, various models have been developed, such as non-linier network programmes (Jordan and Turnquist 1983; Beaujon and Turnquist 1991), multistage dynamic networks (Cheung and Powell 1996), and logistic queuing networks (Powell and Carvalho 1998). The other research direction pursued by researchers later is development of control policies based on traditional inventory control policies (s, Q) and (s, S). Literature that fall into this category include a single-value threshold control policy (Du and Hall 1997), a pull-type decentralised control policy (Hall and Zhong 2002), and steady-state repositioning policy (Köchel, Kunze, and Nieländer 2003). Köchel et al. concluded from the literature that no closed-form solution of optimal empty container/vehicle repositioning is available for the stochastic transportation service systems. Recently, Song (2005) studied a continuous-review two-depot service system and obtained an exact solution of the optimal vehicle redistribution policy by using a uniformisation technique and stochastic dynamic programming.

There is extensive literature on the repositioning of empty containers, called the empty container allocation problem. Crainic, Gendreau, and Dejax (1993a; Crainic et al. 1993b) specifically considered the repositioning of empty containers in a stochastic environment as a mixed integer programme and extended it to a multi-commodity network flow model, which they solved with a Tabu search procedure. Nevertheless, their models charge leasing cost as a fixed amount in one-off mode. Shen and Khoong (1995) then developed a decision support system to optimise container leasing decisions assuming that all demands, schedules, and capacity were deterministic within the planning horizon. In their model, the leasing cost is charged according to the duration of the lease. In the line of stochastic environments, besides the models developed by Crainic, Gendreau, and Dejax (1993a; Crainic et al. 1993b), others include a simulation model (Lai, Lam, and Chan 1995), a two-stage stochastic network model (Cheung and Powell 1996; Cheung and Chen 1998), and a single- and multi-commodity network models (Godfrey and Powell 2002a, 2002b; Topaloglu and Powerll 2004). Lam, Lee and Tang (2007) demonstrated the successful application of an approximate dynamic programming approach in deriving effective operational strategies for the relocation of empty containers. Erera, Morales, and Savelsbergh (2009) developed a robust optimisation framework using time-space networks. In their previous work (Erera, Morales, and Savelsbergh 2005), they also considered integrating repositioning with load assignment decisions. Song (2007) found the optimal stationary policy of empty container repositioning in a periodic-review shuttle service system. Song and Dong (2008) investigated different control policies for empty container management in dynamic and stochastic situations. Simulation was used to evaluate the proposed policies, and a sensitivity analysis was constructed to compare the different policies. In another paper, Dong and Song (2009) considered joint container fleet sizing and empty container repositioning and proposed a simulation-based optimisation approach to simultaneously optimise container fleet size and determine empty repositioning policy. Song and Dong (2011) later also proposed flow balancing-based empty container repositioning in typical shipping service routes. Modern artificial intelligence approaches have also been applied to solve the repositioning of empty containers. In the study of Wong, Yeung and Lau (2009), a novel immunity-based hybrid evolutional algorithm, known as Hybrid Artificial Immune Systems, was proposed to solve the global repositioning of containers. Chou et al. (2010) proposed a mixed fuzzy decisionmaking and optimisation programming model to solve empty container allocation. Their paper also thoroughly reviewed the literature in solving empty container allocation problems. In the last several years, foldable containers were recommended for the shipping industry for alleviating the considerable cost of repositioning of empty containers (Moon, Ngoc, and Koings 2013).

Most of the models for empty container repositioning require a specified planning horizon (Crainic, Gendreau, and Dejax 1993a; Crainic et al. 1993b; Cheung and Powell 1996; Cheung and Chen 1998; Godfrey and Powell 2002a, 2002b; Topaloglu and Powerll 2004). Nevertheless, it was found that the length of the planning horizon had a significant impact on the outcomes of those models under multimodal circumstances – the longer the planning horizon, the more the utilisation of inexpensive and slow transportation modes. Traditional logistics solutions require a tedious data collection effort, and the numerical optimisation could be NP-hard to solve on some occasions (Daganzo 2005; Chou et al. 2010). In his monograph 'Logistic System Analysis', Daganzo (2005) proposed an alternative approach for logistic system planning and analysis. With his method, 'detailed data are replaced by concise summaries, and numerical methods are replaced by analytic models' (1). He claimed that the analytic models can be solved accurately without various pieces of detailed information.

This study applied the methods for designing the strategies for repositioning empty containers in the continental USA. Sensitivity analysis was conducted on the responses of the strategies to critical logistic parameters.

4. Supply and demand assessment

In the context of this study, repositioning of empty containers is a response to the demand for containers in Asia. The following variables must be defined:

- Demand in the USA demand for empty containers in locations in the USA that have export loads to Asia; there is also demand for domestic use.
- Supply in the USA supply of empty containers in the USA is the flow from Asia minus the percentage that needs to be displaced. Even though some containers can be used to transport goods within the USA, they need to be returned to Asia eventually.
- Net supply (NS) and Net demand (ND) in the USA the difference between supply and demand. If supply is higher than demand, there is a NS; otherwise, there is a ND.

Containers in the USA (bound to Asia) can be divided into three types:

- 1. Containers repositioned in the USA to be loaded (including those loaded at the same destination where they were unloaded).
- Empty containers that could not be allocated to a load, which will be transported directly to one of the US ports.
- 3. Containers that could be allocated to a load within the USA

The demand in the USA is the sum of Types 1 and 3. As it is known that only a small proportion of containers can be utilised for loads within the USA for the selected shipping line, it is reasonable to disregard Type 3 at this point. Type 1 is equal to the westbound loads (W), which will determine the demand in the USA. As each westbound load has a specific origin and destination, $W = S_{ij}(W_{ij})$, where *i* is an origin in the USA and *j* is a destination in Asia. Thus, the west-bound demand is DEM = $W = S_{ij}(W_{ij})$. As we are primarily interested in the allocation of empty containers within the USA, it is also important to define demand for each origin *i* in the USA: DEM_i = $S_j(W_{ij})$.

The supply in the USA is the sum of eastbound containers and containers that are already in the USA, minus containers that are being displaced in the USA. We can disregard the containers that are already in the USA if we consider this problem for the long run. As the average life cycle of a container is approximately 10 years (Lopez 2003), we assume that 10% of the containers will be displaced in the USA every year. Although each eastbound load has a specific origin *i* and destination *j*, only the latter is relevant in this study. Therefore, $E = \text{Sum}(E_j)$, and the supply in the USA can be determined by the following equation: $\text{SUP} = 0.9^*E$. We can also determine the supply for each location *j* in the USA: $\text{SUP}_j = 0.9^*S_i(E_{ij})$. It follows that the NS and ND of empty containers in the USA can also be determined for every location *i* in the USA:

$$NS_i = SUP_i - DEM_i$$
, if $SUP_i > DEM_i$,
 $ND_i = DEM_i - SUP_i$, if $DEM_i > SUP_i$.

Detailed data were provided for both eastbound loads (E_{ij}) and westbound loads (W_{ij}) between Asia and North America, divided into four quarters. Data were aggregated and included only major terminals (and not the first and last legs). Locations in Mexico and Canada were disregarded for simplification purposes. From these data, supply, demand, NS, and ND (of empty containers) in the USA were determined based on the above equations.

5. Methodology

Three strategies were proposed: (1) Introducing 'terminals' into the shipping line's network – the investigation of the current empty container management suggests that the introduction of

'terminals' would increase efficiency and reduce overall cost. By adding terminals, road distance can be minimised to the detriment of higher rail distances, which incur lower costs. Each terminal is assigned to an influence area that includes a fixed number of customers. Therefore, customers 'belong' to a specific terminal. To simplify the modelling, terminals were assumed to handle the same number of containers. In this context, NS refers to the difference between supply and demand at a given influence area if supply is higher than demand; ND refers to the same difference should demand be higher than supply (ND = -NS). An influence area and its respective terminal belong to a region, which was chosen so that the demand and supply were slow-varying; (2) Utilising ports on East Coast. Empty containers are currently not sent back to Asia through the East Coast ports due to less frequent schedules and capacity constraints. However, we believe that it is possible to reduce costs by utilising such ports, especially for locations in the East Coast, and (3) Utilising a dedicated fleet of trucks.

Step 1. Preliminary analysis

To account for stochastic variations in our network, the USA was divided into regions for which current supply and demand of containers were approximately uniform in each region.

Step 2. Optimisation

The goal of this step was to develop a logistics cost function that could be minimised in terms of the number of terminals and the headways (decision variables). The cost function has deterministic and stochastic components and was divided into four levels:

- *Local level.* This level concerns costs associated with the movement of containers from a terminal to customers in an influence area, and vice versa. It was assumed that road transportation is the only mode utilised in this level.
- *Regional level*. This level deals with the movement of containers among terminals within the same region. It was assumed that only rail is utilised.
- *Inter-regional level*. This level deals with the movement of containers between regions. It was assumed that the movement of containers between regions is done by rail.
- *National level*. This includes the costs of transporting containers from terminals to seaports by rail.

Deterministic component

(a) Local

Local-l	evel parameters
c_t :	Variable trucking cost (\$/distance. container) \$1/(mile.feu)
c _i :	Inventory cost (\$/time. container) \$0.78/(day feu)
s _t :	Truck speed (distance/time) 1200 miles/day
c _{ht} :	Fixed trucking cost (\$/container) \$50/feu, includes handling at customer and drop-off at terminal
$D_{\rm L}$:	Average local distance (miles)
$A_{\rm R}$:	Region area (miles ²)
$N_{\rm R}$:	Number of terminals in the region
MAX:	Maximum between supply and demand (feu)

The local level involves the costs associated with the movement of containers between a terminal and its customers. Each terminal services one area of customers, which was approximated by a circle. Then, a simplification could then be made for the average distance from a customer to its respective terminal. Assuming a uniform distribution of customers within an influence area, the average distance from each customer to the terminal was determined to be two-thirds of the radius of the area.

$$D_{\rm L} = \frac{2}{3} \cdot (\text{Radius of area}) = \frac{2}{3} \sqrt{\frac{A_{\rm R}}{N_{\rm R} \cdot \pi}}.$$

It has been assumed that handling costs are included in the fixed truck rate, as it was discovered that many costs are collected in contracts made for the outsourced modes of transportation. To model this level of the network, the supply and demand were paired so that the total number of round trips was the maximum of total supply and total demand for each area. It was also assumed that some of the customer locations have simultaneous supply and demand. Thereby, we estimated that 10% of containers do not need to go through terminals.

Breakdown of Average local costs:

Transportation cost =
$$c_t \cdot D_L \cdot 2 \cdot MAX \cdot 0.9$$
,
Handling cost = $c_{ht} \cdot MAX \cdot 0.9$,
Inventory cost = $(c_i \cdot D_L/s_t) \cdot MAX \cdot 0.9$.

(b) *Regional level*

Regio	onal-level parameters		
$c_{\rm r}$:	Variable rail cost with distance (\$/distance. container)	25 cents/(mile.FEU)	
$c_{\rm hr}$:	Fixed rail cost – handling (\$/container)	\$40/FEU	
c_{fr} :	Fixed rail cost per dispatch (\$/dispatch)	\$200/dispatch	
<i>s</i> _r :	Rail speed (distance/time)	600 miles/day	
$c_{\rm fm}$:	Fixed cost of terminals (\$/terminal) {\$8000 for terminals at \$80000 otherwise	APL city	
c _m :	Variable cost of terminal – renting (\$/time container)	1 \$/(day FEU)	
D_{R} :	Average distance between terminals in one region (distance)	miles	
δ_{T} :	Terminal density, i.e. number of terminals in region/area of $1/\text{miles}^2$	region (1/distance ²)	
$H_{\rm R}$:	Headway to transport containers among terminals (time) day	S	
ND _R :	D_R : ND in one area (feu)		
NS _R :	NS in one area (feu)		

At the local level, containers in one area are transported from customers to a single terminal, which, in turn, fulfills the demand of containers within its influence area. The result will be either NS at the terminal (if supply > demand) or ND (demand > supply). The regional-level cost involves the relocation of containers from terminals with NS to terminals with ND.

The distance between terminals in one region (D_R) was calculated based on the assumption that terminals were uniformly distributed. The constant *k* accounts for the fact that a rail line is never a straight line and, in this case, took a value of 1.13 $(2/\sqrt{\pi})$.

$$D_{\mathrm{R}} = k \cdot \delta_{\mathrm{T}}^{-1/2} = k \cdot \sqrt{\frac{A_{\mathrm{R}}}{N_{\mathrm{R}}}}.$$

Costs at the regional level were divided into transportation costs, handling costs, inventory costs, and terminal costs. Transportation was done only by rail and its costs have two components, $c_{\rm fr}$, and $c_{\rm r}$. The first is related to the fixed cost per dispatch; the second represents the portion that varies with distance. Note that the number of containers that have to be transported among terminals in one region is the ND (the demand that cannot be satisfied by the supply) in the region, as containers will not be relocated unless they are needed somewhere. The handling cost involves the handling of each container from the rail storage area to the train on both origin and destination rail terminals. Finally, inventory costs include both stationary and pipeline inventories. For the first, it is assumed that, on average, containers spend half headway at the origin rail terminal and half headway at the destination rail terminal.

Transportation cost =
$$c_{\rm fr} \cdot \frac{1}{H_{\rm R}} \cdot N_{\rm R} + c_{\rm r} \cdot D_{\rm R} \cdot {\rm ND}_{\rm R}$$
,
Handling cost = $c_{\rm hr} \cdot {\rm ND}_{\rm R}$,
Inventory cost = $c_{\rm i} \cdot \left(H_{\rm R} + \frac{D_{\rm R}}{s_{\rm r}}\right) \cdot {\rm ND}_{\rm R}$.

To compute the terminal costs, it was assumed that terminals were located at rail yards. As soon as containers become available at customer locations, they are shipped to the terminals where they are stored until the next train dispatch. This assures that containers are always available at the terminals and enables economies of scale in dispatching a group of containers at once. It was assumed that each terminal serves the same amount of containers, which was determined by the maximum between total supply and total demand in one region divided by the number of terminals.

It was assumed that cities should receive priority as a terminal, and the cost there would be about \$8000 to rent a storage area. If the optimal number of terminals exceeded 56, the cost of opening a new terminal would then be \$80,000. In addition, a rental cost was added. This was calculated by multiplying the rate per container times the maximum space needed by any given terminal.

(c) Inter-regional level

Inter	-regional-level parameters
$H_{\rm I}$:	Headway to transport containers between regions (days)
D_{I} :	Average inter-regional distance (miles)
$L_{\rm E}$:	Horizontal length of East Region (miles)
L_{W} :	Horizontal length of West Region (miles)
NDI	ND in one region (e.g. overall ND in the East region) (feu)
NS _I :	NS in one region (feu)

After containers are relocated among terminals within the same region, the NS and ND at the regional level are assessed for both the East and West regions. It is intuitive that relocation at this level happens only if at least one region has an overall ND. In this specific case, both East and West have overall NS; therefore, this step was not necessary. However, this study was intended for more general cases, so the methodology for this step is as follows.

The average inter-regional distance was calculated by picturing a virtual regional terminal approximately in the middle of each of the two regions (Figure 3). As both regions – East and West – have the same vertical length, only the horizontal separation was taken into account.

$$D_{\rm I} = \frac{L_{\rm E} + L_{\rm W}}{2}.$$

Costs at the inter-regional level were divided into transportation, handling, and inventory costs, which have the same form described at the regional level. Therefore:

Transportation cost = $c_{\rm fr} \cdot \frac{1}{H_{\rm I}} + c_{\rm r} \cdot D_{\rm I} \cdot {\rm ND}_{\rm I}$, Handling cost = $c_{\rm hr} \cdot {\rm ND}_{\rm I}$, Inventory cost = $c_{\rm i} \cdot \left(H_{\rm I} + \frac{D_{\rm I}}{s_{\rm r}}\right) \cdot {\rm ND}_{\rm I}$,



Figure 3. Average inter-regional distance.

(d) National level

National-level parameters

$D_{\rm N1}$:	Average distance between terminals to the ports in the same region (miles)
$D_{\rm N2}$:	Average distance between terminals to the ports in the other region (miles)
$H_{\rm N1}$:	Headway to transport containers to terminals in the same region (days)
H_{N2} :	Headway to transport containers to terminals in the other region (days)
$L1_R$:	Horizontal length of the same region (miles)
$L2_{R}$:	Horizontal length of the opposite region (miles)
$V_{\rm R}$:	Vertical length in one region (miles)
SP1 _R :	Supply that goes to ports in the same region (feu)
SP2 _R :	Supply that goes to ports in the opposite region (feu)

The national level is responsible for the relocation of remaining containers in the regions to the seaports on both West and East coasts after the regional ND is fulfilled. There are additional capacity constraints on East Coast ports due to fewer ship schedules bound to Asia. For simplification purposes, it was assumed that all ships provide the same proportion of their capacity to empty containers, and also the same proportion at each port that they detour before they depart to Asia. From the analysis, ships at East Coast ports can carry about 40% of the containers that originate in the East Region; the remaining volume has to be transported to ports on the West Coast.

For the West Region, due to the low density of containers in the Midwest, the average horizontal distance to the West Coast ports is one-third of the region's horizontal length. After fulfilling the ND of some of the terminals, the East Region was divided into two sub-regions according to the capacity proportion. Containers in Sub-region 1 were transported to East Coast ports with average horizontal distance equal to half of the sub-region's horizontal length. Containers in Sub-region 2 were transported to West Coast ports, and the average distance was half of the sub-region's horizontal length plus the horizontal length of the West Region. There are four ports in each region. We assumed that the average vertical distance for each container to the ports is one-eighth of the vertical length. Final numbers are presented in Figure 4.

$$D_N 1 = L 1_{\rm R} \cdot q \cdot i + 1/8 \cdot V_{\rm R} \quad i = \begin{cases} \frac{1}{2} & \text{for east region} \\ \frac{1}{3} & \text{for west region} \end{cases}$$

where q is the proportion of containers transported to the ports in the same region.

$$D_{N2} = (1-q) \cdot (1-i) L 1_{\rm R} + L 2_{\rm R} + \frac{1}{8} \cdot V_{\rm R}.$$

For the containers transported to ports in the same region:

Transportation cost $1 = c_{fr} \cdot \frac{1}{H_{N1}} \cdot N_{N1} + c_r \cdot D_{N1} \cdot SP1_R$, Handling cost $1 = c_{hr} \cdot SP1_R$, Inventory cost $1 = c_i \cdot \left(\frac{H_{N1}}{H_{N1}} + \frac{D_{N1}}{H_{N1}}\right) \cdot SP1_R$.

ventory cost
$$1 = c_i \cdot \left(\frac{H_{N1}}{2} + \frac{D_{N1}}{s_r}\right) \cdot \text{SP1}_{\text{F}}$$



Figure 4. Average distances to ports.

Note that the stationary inventory involves only half of the headway (inventory time at the port is not considered).

Similarly, for the containers transported to the ports in the other region:

Transportation cost 2 =
$$c_{\rm fr} \cdot \frac{1}{H_{N2}} \cdot N_{N2} + c_{\rm r} \cdot D_{N2} \cdot \text{SP2}_{\rm R}$$
,
Handling cost 2 = $c_{\rm hr} \cdot \text{SP2}_{\rm R}$,
Inventory cost 2 = $c_{\rm i} \cdot \left(\frac{H_{N2}}{2} + \frac{D_{N2}}{s_{\rm r}}\right) \cdot \text{SP2}_{\rm R}$.

Stochastic component

As the supply and demand for containers are not deterministic, stochastic variations were accounted for in this part. In the previous section, the expected number of containers was relocated within an area (local level), among areas in one region (regional level), and to the seaports (national level). This assured the whole country would be balanced. In other words, the NS/ND in all locations (e.g. seaports, terminals, and customers) can be considered zero after the deterministic portion is solved.

Note that the deterministic component contains a term to account for the effect of demand and supply variance in the terminal renting cost, so this was not included here. Thus, the stochastic component is concerned only with the relocation cost of containers among terminals.

The total container-distance travelled per terminal (p) was determined from a function formulated by Daganzo and Smilowitz (2004). The formula assumed that a region with an expected density of terminals (δ_R) had an NS/ND with zero mean and standard deviation σ , and N_R terminals.

$$p = \sigma \cdot \delta_{\rm R}^{-1/2} \cdot f(N_{\rm R}) = \sigma \cdot \delta_{\rm R}^{-1/2} \cdot \left[0.43 + 0.02 \log_2(N_{\rm R}) \right].$$

The total cost has a similar form to that described in the regional level of the deterministic part. The number of containers in each terminal dispatch is $\sigma^* H_R$, so it follows that:

Transportation cost (regional) =
$$c_{\rm fr} \cdot \frac{1}{H_{\rm R}} \cdot N_{\rm R} + c_{\rm r} \cdot P_{\rm R} \cdot N_{\rm R}$$
,
Handling cost = $c_{\rm hr} \cdot \sigma \cdot N_{\rm R}$,
Inventory cost = $c_{\rm i} \cdot \left(H_{\rm R} \cdot \sigma + \frac{p_{\rm R}}{s_{\rm r}}\right)$.

Deterministie eomponent	
Local level	$c_{t} \cdot D_{L} \cdot 2 \cdot MAX \cdot 0.9 + c_{ht} \cdot MAX \cdot 0.9 + \frac{c_{i} \cdot D_{L}}{s_{t}} \cdot MAX \cdot 0.9$
Regional level	$c_{\rm fr} \cdot \frac{1}{H_{\rm R}} \cdot N_{\rm R} + c_{\rm r} \cdot D_{\rm R} \cdot {\rm ND}_{\rm R} + c_{\rm hr} \cdot {\rm ND}_{\rm R} + c_{\rm i} \cdot \left(H_{\rm R} + \frac{D_{\rm R}}{s_{\rm r}}\right) \cdot {\rm ND}_{\rm R}$
	$+c_{\rm fm} \cdot N_{\rm R} + c_{\rm m} \cdot ({\rm MAX} + 3\sigma) \cdot H_{\rm R}$ $1 \qquad \qquad D_{\rm I}$
Inter-regional level	$c_{\rm fr} \cdot \frac{H_{\rm I}}{H_{\rm I}} + c_{\rm r} \cdot D_{\rm I} \cdot ND_{\rm I} + c_{\rm hr} \cdot ND_{\rm I} + c_{\rm i} \cdot (H_{\rm I} + \frac{1}{s_{\rm r}}) \cdot ND_{\rm I}$
National level	$c_{\rm fr} \cdot \frac{1}{H_{N1}} \cdot N_{N1} + c_{\rm r} \cdot D_{N1} \cdot \operatorname{SP1}_{R} + c_{\rm hr} \cdot \operatorname{SP1}_{R} + c_{\rm i} \cdot \left(\frac{H_{N1}}{2} + \frac{D_{N1}}{s_{\rm r}}\right) \cdot \operatorname{SP1}_{R}$
	$+c_{\rm fr} \cdot \frac{1}{H_{N2}} \cdot N_{N2} + c_{\rm r} \cdot D_{N2} \cdot \text{SP2}_{\rm R} + c_{\rm hr} \cdot \text{SP2}_{\rm R} + c_{\rm i} \cdot \left(\frac{H_{N2}}{2} + \frac{D_{N2}}{s_{\rm r}}\right) \cdot \text{SP2}_{\rm R}$
Stochastic component	$c_{\mathrm{fr}} \cdot \frac{1}{H_{\mathrm{R}}} \cdot N_{\mathrm{R}} + c_{\mathrm{r}} \cdot P_{\mathrm{R}} \cdot N + c_{\mathrm{hr}} \cdot \sigma \cdot N_{\mathrm{R}} + c_{\mathrm{i}} \cdot (H_{\mathrm{R}} \cdot \sigma + \frac{P_{\mathrm{R}}}{s_{\mathrm{r}}})$

Table 2. Total cost function.

Deterministic component

Table 3. Optimisation results.

Optimisation results	East region	West region	Total
Total cost	58,098,500	59,667,700	117,766,200
Number of terminals	53	40	93
Regional headway (among terminals) (days)	5	4	
National headway – to port in the same region (days)	17	12	
National headway – to port in the opposite region (days)	11	-	

The cost function included all costs in both deterministic and stochastic parts. Table 2 provides a summary of the cost function:

Constraints

The demand for empty containers in Asia did not have to be considered as a constraint. As empty containers are produced in Asia, they can be purchased there if it is more cost-effective. Even though rail capacity can be an issue in practice, it was disregarded for simplification purposes. Port capacity was taken into account in order to prioritise the shipment of containers to different ports. We also assumed an infinite supply of trucks and of storage area at seaports and rail terminals. The number of terminals in the West Region was constrained at 40, as it is the number of terminals served by the major rail companies.

Optimisation results

Regions were permitted to have different number of terminals and headways, so, in fact, there are two decision variables representing the number of terminals in each region and the three types of headways mentioned. The optimisation model yielded the results that are given in Table 3. Further analysis of such results is included in Section 5.

Step 3: Network design

Step 2 provides the optimal number of terminals in both regions. This step is responsible for choosing the location of such terminals and determining their influence areas.

Actual location of terminals

Terminals for repositioning containers will be located at existing rail terminals, where yard space will be rented as needed. It was assumed that there would always be yard space available. From a list of rail terminals provided by the major rail companies, cities were ranked by the following factors (in order): (1) whether the shipping line currently has operations in the city (because there is an implicit penalty for opening a new facility), (2) number of rail routes that go through the city, and (3) number of rail companies that serve the city.

The optimal number of terminals in the East, $N_E^* = 53$, matched up precisely with the number of leading cities on the ranked list. In the initial run of the optimisation model, the output number of terminals for the West Region was 78. However, this value became unreasonable when compared to the number of existing rail terminals. A restriction was added to the model to reduce the number of terminals within the range of feasible rail terminals. (Note: In the later section on sensitivity analysis, it can be seen that this restriction has a rather insignificant effect on the total costs.) The final result for the optimal value of terminals in the West was $N_W^* = 40$.

Determination of influence areas

Based on the actual location of terminals, it was assumed that each customer was allowed to send containers to its nearest terminal. The specific algorithm to assign influence areas to terminals assumed that demand was given by a function per square mile. Thus, we considered each 1×1 square mile a fictional customer with demand for a certain amount of containers. The Voronoi Tesselation method was used to assign each fictitious customer to a terminal and thus determine the influence areas (Voronoi Tessellation 2003). Influence areas can be thought of as Voronoi regions, where all interior points are closer to the corresponding generator, or terminal, than to any other. Then, the Voronoi Tessellation is the set of Voronoi regions where no pairs of Voronoi regions have a common point. The distance is $d(z, z_i) = ||z - z_i||$, where z is a terminal location coordinates and z_i is a 1 mile \times 1 mile region representing a fictitious customer. Average distances were calculated from every $d(z, z_i)$.

Step 4: Cost calculation

By determining the actual location of terminals, total logistics costs will be slightly different from the optimised costs. The same four levels considered were evaluated to calculate the actual costs of the proposed strategies:

(a) Local level

Once the influence areas were determined, local-level costs (from customers to terminals and vice-verse within an influence area) could be calculated.

Section 3 demonstrated how supply and demand were determined for each data point provided. Based on such information, each customer's demand and supply was estimated. For the purpose of determining the customer demand/supply density only, the 56 data points were regarded as terminals that handled containers in their own influence area. To allocate customers to terminals (to establish influence areas), it was determined that each customer was allowed to send containers to their nearest terminal. Assuming customers are uniformly located in each area and all the customers within the same area are identical, the customer demand/supply density for each area could be obtained.

Based on this density function and on the location of terminals, the influence areas (e.g. total area, shape, and container average travel distance) could be determined through the same methodology. By combining the demand and supply of each customer within an influence area, the number of containers handled by the terminal was calculated. The local cost of transporting containers

Cost item	Baseline	Optimisation	Actual
Trucking costs	\$74,567,838	\$42,284,200	\$42,793,671
Rail costs	\$73,051,908	\$45,277,987	\$52,414,260
Inventory costs	\$3,413,542	\$1,024,332	\$5,185,021
Handling costs	\$9,223,808	\$24,324,150	\$24,324,150
Terminal costs	\$0	\$4,825,590	\$4,972,441
Total costs	\$160.257.096	\$117,736,259	\$129.689.443
% Savings compared to baseline	,,	26.53%	19.07%

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between customers and terminals was obtained as a function of the average travel distance and the volume of containers handled by a terminal.

(c) Regional, inter-regional, and national level

Supply and demand values for each terminal were calculated by aggregating the supply and demand densities over each influence area. By regarding ports as terminals with ND, balancing containers within the whole country was captured by a minimal cost network flow problem. The regional, inter-regional, and national-level costs were determined. In this specific case, inter-regional costs were not applicable, as there were no flows between both regions. Ports' demand was determined according to their capacity.

Cost calculation results

Based on the actual location of terminals and their respective influence areas, new costs were calculated. Table 4 compares such results with baseline and optimised costs and reflects that significant savings (19.07%) can still be realised by applying the proposed strategies. Additionally, the main differences between the actual and the optimised costs rely on the rail costs. This is true, as rail distances are higher once the actual locations of terminals is determined, and terminals in the East Region are forced to ship the remaining empty containers to West Coast ports due to capacity constraints on the ports in the East Coast. Actual inventory costs are higher than optimised inventory costs, as the maximum headway between the regional and the national was utilised to simplify the calculations. Both handling and inventory costs remain fairly constant, as they do not depend on the terminal locations.

Sensitivity analysis

The optimisation results from the previous section provided an optimal solution by minimising total logistics costs based on several parameters and constraints. To determine how robust the logistics cost function is to these parameters, constraints, and strategies, sensitivity analyses were performed.

At the strategic level, two scenarios were considered: (1) the exclusion of East Coast ports; thus, all containers are shipped to West Coast ports; (2) the use of a dedicated truck fleet. At the tactical level, the constraint on the number of terminals is relaxed. Finally, at the operational level, the following parameters were modified by -10% and +10%: unit inventory cost, variable rail cost related to the distance traveled, fixed rail cost per dispatch, rail handling cost per container, and terminal costs. A few other parameters were also modified, including container volume, existing terminal cost, and new terminal cost. Each variation was calculated separately by running the optimisation model with different parameters, and the output was always compared to the original optimised cost.



Figure 5. Impact of unrestricted number of terminals on total cost.

By shipping all containers to the West Coast, the distance travelled increased and total costs went up by 15.35% in this model. In other words, from the 26.53% savings, 15.35% are due to the use of East Coast ports, and the remaining 11.18% are due to the use of terminals.

As trucking costs represent more than 35% of total logistics costs, the use of a dedicated truck fleet was evaluated. The analysis indicates that 361 trucks would be necessary to manage such operations (215 for the West Region and 146 for the East Region). Due to the magnitude of the operations of a 361-fleet of trucks, personnel must be hired to manage such operations. An annual cost of approximately \$4 million was estimated. By adding this cost to the fixed costs per truck with the variable cost per distance, the new total cost decreased by 21%. Even if labour and fuel costs are increased by 20% to be conservative, total cost still decreases by over 16% compared to the optimisation result. This indicates the potential benefits of utilising a dedicated fleet to manage the repositioning of empty containers in the USA. Further analysis is necessary to determine the practical implications of such a strategy.

As previously mentioned, the optimisation results indicated that the West Region should have 78 terminals. Since the major rail companies only have strong operations on 40 cities on the West Region, it was decided to restrict the number of terminals to 40. Even if the number of terminals in the West Region is unrestricted, the total cost increases by less than 3%, as indicated by Figure 5. Therefore, the restriction in the number of terminals to 40 (in the West Region) does not have significant implications for the total logistics costs.

Cost coefficients variation

The impact of changing inventory costs, fixed rail costs, rail handling costs, and terminal costs does not have a significant impact on total logistics costs, number of terminals, or headways. The only exception was the variable rail cost, which changed total logistics costs by $\pm 4\%$. Figures 6–8 illustrate such trend.

Volume variation

As expected, total logistics costs vary proportionally with a change in the supply of empty containers. With an increase in the supply by +10% and 20%, the total cost increased by 9.18% and



Figure 6. Impact of cost coefficients on total cost.

18.32%, respectively. As supply decreased by 10% and 20%, total costs decreased by 9.22% and 18.49%. As supply is much higher than demand, a change in the latter did not affect the total cost significantly.

Influence of strategies

Based on the sensitivity analyses, the influence of each of the three strategies was assessed: (1) use of East Coast ports, (2) use of terminals, and (3) use of dedicated fleet. The original optimised results account for both Strategies 1 and 2. The influence of Strategy 1 can be determined by avoiding the use of East Coast ports, thus shipping all empty containers to ports on the West Coast. The influence of Strategy 2 is the difference between the original optimised results and using Strategy 1 alone. The impact of Strategy 3 is the difference between the scenario where dedicated trucks were utilised and the original optimised results (Strategies 1 and 2 together). Table 5 presents the comparison between these three strategies against the baseline.

The projected use of all three strategies is promising. East Coast ports can be utilised without significant implementation costs and still yield savings of 11.3%. Terminals bring more structural changes to the current network, which is more challenging to implement. However, this brings higher savings of 15.2%. The use of a dedicated fleet of trucks is the strategy with the highest associated risk, as significant capital costs would have to be incurred. However, potential savings of 15.4% are also encouraging.

Further analysis is necessary to implement any of these strategies, as there might be additional practical implications not considered in this study. This is especially important for Strategies 2 and 3.



Figure 7. Impact of cost coefficients on number of terminals.



Figure 8. Impacts of cost coefficients on headways.

	Total cost	Savings compared to baseline
Baseline	\$160,257,096	
Original optimised results (Strategies 1 and 2)	\$117,736,259	
Transport all containers to the West Coast	\$135,841,900	
Use dedicated fleet	\$93,108,600	
Strategy 1: Use of East Coast ports	\$18,105,641	11.3%
Strategy 2: Use of terminals	\$24,415,196	15.2%
Strategy 3: Use of dedicated fleet	\$24,627,659	15.4%

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Table	<u>٦</u>	Strategy	comparison
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Future steps

The problem of repositioning empty containers has great depth. To get a more realistic view of the effects of our suggestions, the following further investigations should be considered. First, it is important to assess the practical implications of implementing a terminal system to the reallocation of containers. These are difficult to model but are nonetheless significant. Second, more comprehensive research should be done on the costs of designing, implementing, and operating a dedicated truck fleet. Our estimates for these values are rough, but the possible savings are enough to warrant additional examination. Third, our model assumes that a rail terminal can use any headway. However, rail transportation is restricted to schedules. Synchronising the reallocation of containers to involve rail (and possibly port as well) schedules would provide a more practical idea of the entire system.

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